 MST–018

M. SC. (APPLIED STATISTICS) (MSCAST)

Term-End Examination

June, 2025

MST-018 : MULTIVARIATE ANALYSIS

Time : 3 Hours *Maximum Marks : 50*

Note : *(i) Question No.* ***1*** *is compulsory.*

(ii) Attempt any ***four*** *questions from the remaining question nos.* ***2*** *to* ***6****.*

(iii) Use of scientific (non-programmable) calculator is allowed.

(iv) Symbols have their usual meanings.

1. State whether the following statements are True *or* False. Give reasons in support of your answers : 5×2=10

   (a) The eigen values of a positive definite matrix are less than equal to zero.

(b) Multiple correlation is the minimum correlation between the linear combination of the components of *x* and the linear combination of the components of *y*.

(c) $A = \begin{pmatrix} -1/\sqrt{2} & 1/\sqrt{2} \\ 1/\sqrt{2} & 1/\sqrt{2} \end{pmatrix}$ is an orthogonal and idempotent matrix.

(d) A factor model postulates that a random vector $\underset{\sim}{X}$ is linearly dependent upon a few observable common factors.

(e) If $\underset{\sim}{X} \sim N_3(\underset{\sim}{\mu}, \Sigma)$ with $\underset{\sim}{\mu} = \begin{pmatrix} 2 \\ 1 \\ -1 \end{pmatrix}$ and

$\Sigma = \begin{pmatrix} 5 & 2 & 0 \\ 2 & 2 & -1 \\ 0 & -1 & 4 \end{pmatrix}$, then the distribution of $\left(\underset{\sim}{X} - \underset{\sim}{\mu}\right)' \Sigma^{-1} \left(\underset{\sim}{X} - \underset{\sim}{\mu}\right)$ is non-central Chi-square with 3 degrees of freedom.

2. Let $\underset{\sim}{X}_{P\times 1} \sim N_P\left(\underset{\sim}{\mu}, \Sigma\right)$. Also, let $\underset{\sim}{X}$, $\underset{\sim}{\mu}$ and Σ be partitioned as :

$$\underset{\sim}{X}_{P\times 1} = \begin{pmatrix} \underset{\sim}{X}^{(1)}_{K\times 1} \\ \underset{\sim}{X}^{(2)}_{(P-K)\times 1} \end{pmatrix}, \quad \underset{\sim}{\mu}_{P\times 1} = \begin{pmatrix} \underset{\sim}{\mu}^{(1)}_{K\times 1} \\ \underset{\sim}{\mu}^{(2)}_{(P-K)\times 1} \end{pmatrix}$$

and

$$\Sigma_{P\times P} = \begin{pmatrix} \Sigma_{11_{K\times K}} & \Sigma_{12_{K(P-K)}} \\ \Sigma_{12_{(P-K)\times K}} & \Sigma_{22_{(P-K)\times(P-K)}} \end{pmatrix}$$

Then prove that the conditional distribution :

$$\underset{\sim}{X}^{(1)} \mid \underset{\sim}{X}^{(2)} = \underset{\sim}{X}^{(2)} \sim N_K$$

$$\left(\underset{\sim}{\mu}^{(1)} + \Sigma_{12}\Sigma_{22}^{-1}\left(\underset{\sim}{X}^{(2)} - \underset{\sim}{\mu}^{(2)}\right), \Sigma_{11.2}\right),$$

where $\Sigma_{11.2} = \Sigma_{11} - \Sigma_{12}\,\Sigma_{22}^{-1}\,\Sigma_{21}$.

Also, let $\underset{\sim}{X} = \begin{pmatrix} \underset{\sim}{X}^{(1)} \\ \underset{\sim}{X}^{(2)} \end{pmatrix} \sim N_4\ (\underset{\sim}{\mu}, \Sigma)$,

where $\underset{\sim}{\mu} = \begin{pmatrix} -4 \\ 1 \\ \hline 4 \\ 0 \end{pmatrix}$ and $\Sigma = \left(\begin{array}{cc|cc} 2 & 0 & 1 & 0 \\ 0 & 2 & 2 & 0 \\ \hline 1 & 2 & 6 & 1 \\ 0 & 0 & 1 & 1 \end{array}\right)$

Then find $E\left(\underset{\sim}{X}^{(1)} \mid \underset{\sim}{X}^{(2)} = \underset{\sim}{X}^{(2)}\right)$ and $\text{Cov}\left(\underset{\sim}{X}^{(1)} \mid \underset{\sim}{X}^{(2)} = \underset{\sim}{x}^{(2)}\right)$. 10

3. If :

$$A = \begin{pmatrix} 3 & -1 & 0 \\ -1 & 2 & 1 \\ 0 & 1 & 3 \end{pmatrix},$$

then :

(i) Obtain the square root matrix corresponding to the matrix A and verify that $A^{1/2} A^{1/2} = A$. 5

(ii) Determine the first principal component of matrix A and the proportion of the total variability that is explains. 5

4. If : a

$$\underset{\sim}{X} \sim N_4\left(\underset{\sim}{\mu}, \Sigma\right)$$

with

$$\underset{\sim}{\mu} = \begin{pmatrix} 1 \\ 3 \\ 2 \\ -1 \end{pmatrix} \text{ and } \begin{pmatrix} 5 & 0 & 2 & 0 \\ 0 & 4 & 0 & -1 \\ 2 & 0 & 3 & 0 \\ 0 & -1 & 0 & 2 \end{pmatrix},$$

then :

(i) Check whether the variance-covariance matrix Σ is positive definite or not. 2

(ii) Obtain the marginal distribution of $\begin{pmatrix} X_1 \\ X_3 \end{pmatrix}$ and $\begin{pmatrix} X_2 \\ X_4 \end{pmatrix}$. 4

(iii) Comment on the independence of the subvectors $\begin{pmatrix} X_1 \\ X_3 \end{pmatrix}$ and $\begin{pmatrix} X_2 \\ X_4 \end{pmatrix}$. 4

5. Define principal component analysis. Write the stepwise procedure for obtaining the principal components. 10

6. (a) Define Hotelling's T^2 and mention *two* applications of it. Also, give the proof of *one* of them. 5

(b) Define clustering. Differentiate between the single linkage and complete linkage method of clustering. 5

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