# M. SC. (APPLIED STATISTICS) (MSCAST)

### **Term-End Examination**

June, 2025

## MST-021 : CLASSICAL AND BAYESIAN INFERENCE

Time: 3 Hours Maximum Marks: 50

Note: (i) Question No. 1 is compulsory.

- (ii) Attempt any four questions from the remaining Question Nos. 2 to 6.
- (iii) Use of Scientific Calculator (nonprogrammable) is allowed.
- (iv) Symbols have their usual meanings.

- 1. State whether the following statements are True or False. Give reasons in support of your answer:  $5\times2=10$ 
  - (a) If  $\tilde{\mu}_1$  and  $\tilde{\mu}_2$  represent the median of two independent populations, then for testing the hypothesis:

 $H_0\colon \tilde{\mu}_1 \geq \tilde{\mu}_2 \text{ against } H_1\colon \tilde{\mu}_1 < \tilde{\mu}_2 \,,$ 

we use Wilcoxon matched-pair signedrank test.

- (b) In Bayesian approach, we assume the parameter as a constant.
- (c) The Rao-Blackwell theorem enables us to obtain minimum variance unbiased estimator through complete statistic.
- (d) A test which is at least as powerful as any other test of the same size is called most powerful test.
- (e) For testing the hypothesis whether two attributes are independent or not, we use K-S test.

### A-205/MST-021

2. The magnitude of the earthquakes recorded in a region, modelled as an exponential distribution with parameter  $\theta$  whose p.d.f. is given by:

$$f(x) = \theta e^{-\theta x} \quad ; \quad x > 0, \theta > 0$$

If a seismologist wants to test the hypothesis:

$$H_0:\theta=\theta_0 \ against \ H_1:\theta=\theta_1, \left(\theta_1<\theta_0\right),$$
 then derive the test.

3. The following data give the sales of 7 models of mobiles at four different stores. The sales of each mobile (in number of mobiles sold) from each store are given as follows:

| Store A | Store B | Store C | Store D |
|---------|---------|---------|---------|
| 58      | 74      | 35      | 78      |
| 55      | 57      | 51      | 85      |
| 38      | 65      | 41      | 62      |
| 63      | 48      | 52      | 75      |
| 41      | 83      | 54      | 87      |
| 50      | 61      | 53      | 57      |
| 43      | 68      | 57      | 66      |

Test whether there is a significant difference in the sales of the four stores by using Kruskal-Wallis test at 1% level of significance (Given:  $\chi^2_{(3),0.01} = 11.34$ ).

4. If the reduced weight (in kg) after a diet plan follows an exponential distribution with parameter θ whose p.d.f. is given as:

$$f(x) = \theta e^{-\theta x}; \quad x \ge 0, \theta > 0,$$

then: 7+3

- (i) Find the Cramer-Rao lower bound for the variance.
- (ii) Also find UMVUE of  $\frac{1}{\theta}$ .
- 5. The number of customer arrivals at a restaurant, follows Poisson distribution, whose p.m.f. is given as follows:

$$P[X=x] = \frac{e^{-\lambda}\lambda^x}{x!}; x = 0,1,2,\dots,\lambda > 0$$

### A-205/MST-021

It is assumed that the arrival rate  $(\lambda)$  of the customers is a random variable and has gamma prior whose p.d.f. is given as follows:

$$f(\lambda) = \frac{b^a}{\overline{a}} e^{-b\lambda} \lambda^{a-1}; \quad a, b > 0$$

If  $X_1, X_2, \dots, X_n$  represent the number of customer arrivals, then show that gamma is a conjugate distribution for the Poisson distribution.

6. (a) The following data give the number of road accidents that occurred during the various days of a week:

| Day  | No. of Accidents |  |
|------|------------------|--|
| Mon. | 14               |  |
| Tue. | 15               |  |
| Wed. | 8                |  |
| Thu. | 20               |  |
| Fri. | 11               |  |
| Sat. | 9                |  |
| Sun. | 14               |  |

Test whether the accidents are uniformly distributed over the week at 5% level of significance. (Given :  $\chi^2_{(6),0.05} = 12.59)\,. \qquad \qquad 6$ 

(b) Describe OC and ASN function. 4

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