No. of Printed Pages : 5 **MST–022**

M. SC. (APPLIED STATISTICS)

(MSCAST)

Term-End Examination

June, 2025

MST-022 : LINEAR ALGEBRA AND MULTIVARIATE CALCULUS

Time : 3 Hours *Maximum Marks : 50*

Note : *(i) Question No.* ***1*** *is compulsory.*

(ii) Attempt any ***four*** *questions from the remaining Question Nos.* **2** to **6**.

(iii) Use of Scientific calculator (non-programmable) is allowed.

(iv) Symbols have their usual meanings.

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1. State whether the following statements are true *or* false. Give reasons in support of your answers : 5×2=10

(a) The number of critical points of the function :

$$f(x,y)=x^3+y^3-3xy+625$$

is 2.

(b) The directional derivative of the function :

$$f(x,y,z)=x^3+y^2+z^2$$

at point (1, 0, 1) in the direction of the vector (4, 3, 0) is 18/5.

(c) Consider a matrix A of order 5 × 7 such that its rank is 3. The dimension of the null space of A is 2.

(d) Let W_1 and W_2 be two subspaces of a vector space V of dimension 10 such that dim (W_1) = 4, dim (W_2) = 5 and $5 \leq \dim (W_1 + W_2) \leq 9$. Then dim $(W_1 \cap W_2)$ lies between 0 and 4.

(e) The equation of the level surface of the function $f(x,y,z)=\sqrt{y^2-xz}+3y$ through the point (1, 4, 7) is $\sqrt{y^2-xz}+3y=15$.

2. (a) If $A=\begin{bmatrix}1 & 0 & 2\\ -1 & 1 & 3\\ 0 & 0 & 2\end{bmatrix}$, then find algebraic and geometric multiplicities of the smallest eigen value of matrix A. 4

(b) Consider a basis $B=\{(1,1,0),(1,2,0),(0,1,2)\}$ of the vector $\mathbf{R}^3$. Then using the Gram-Schmidt process, obtain an orthonormal basis of $\mathbf{R}^3$ corresponding to B. 6

3. (a) Find the least square solution of the over determined linear system : 5

$$\begin{pmatrix}1 & 1\\ 2 & -1\\ -2 & 4\end{pmatrix}\begin{pmatrix}x_1\\ x_2\end{pmatrix}=\begin{pmatrix}1\\ 2\\ 7\end{pmatrix}$$

(b) If :

$$u = x + y^2 + z^3,$$

$$v = xyz$$

$$w = x^2 - yz$$

then at the point $(x, y, z) = (1,1,1)$, find the value of $\frac{\partial(u,v,w)}{\partial(x,y,z)}$. 5

4. Let M denote the set of all 2 × 2 real symmetric matrices. Show that M is a vector space over **R**, under the vector addition and scalar multiplication, defined as follows :

$$\begin{bmatrix} a & b \\ b & c \end{bmatrix} + \begin{bmatrix} d & e \\ e & f \end{bmatrix} = \begin{bmatrix} a+d & b+e \\ b+e & c+f \end{bmatrix}$$

$$\alpha \begin{bmatrix} a & b \\ b & c \end{bmatrix} = \begin{bmatrix} \alpha a & \alpha b \\ \alpha b & \alpha c \end{bmatrix}$$

where $a, b, c, d, e, f, \alpha \in \mathbf{R}$. 10

5. (a) Consider the function

$$f(x, y) = \sin(\pi x - x^2 y).$$

Find the Hessian matrix H_f that is associated with the second order term in the Taylor's expansion of f around $(1, \pi)$. 5

(b) Find the first four non-zero terms of the Taylor's expansion for the function : 5

$f(x) = e^x + x + \sin x$; about $x = 0$.

6. (a) If $w = xy + z^2$, $x = t^2 e^s$, $y = t \cos s$ and $z = s \sin t$, then find the value of $\frac{\partial w}{\partial t}$ at the point $s = 0, t = \pi$. 4

(b) Show that the function :

$$f(x, y) = \begin{cases} \frac{x^2 y}{x^4 + y^2}, & (x, y) \neq (0, 0) \\ 0, & (x, y) = (0, 0) \end{cases}$$

is not continuous at origin but its partial derivatives f_x and f_y exist at (0, 0). 6

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