No. of Printed Pages : 6 **MSTE-012**

M. SC. (APPLIED STATISTICS)

(MSCAST)

Term-End Examination

June, 2025

MSTE-012 : STOCHASTIC PROCESSES

Time : 3 Hours *Maximum Marks : 50*

Note : *Question No.* ***1*** *is compulsory. Attempt any* ***four*** *questions from the remaining question nos. 2 to 6. Use of scientific (non-programmable) calculator is allowed. Symbols have their usual meanings.*

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1. State whether the following statements are True *or* False. Give reasons in support of your answers : 5×2=10

(a) If parametric space 'T' is discrete and state space 'S' is continuous, then it is a continuous stochastic process.

(b) The difference of two independent Poisson processes $X_1(t)$ and $X_2(t)$ with the parameters $\lambda_1 t$ and $\lambda_2 t$, respectively, is a Poisson process with parameter $(\lambda_1 - \lambda_2)t$.

(c) The average waiting time of a customer in the queue $E(W_q)$ is (λ/μ) times of the average waiting time of a customer in the system $E(W_s)$.

(d) A stationary process $Y_t = X_t + \mu$ is said to be an Autoregressive process, if :

$$X_t = \alpha_1 X_{t-1} + \alpha_2 X_{t-2}^2 + \ldots\ldots + \alpha_p X_{t-p}^p + a_t$$

(e) If P is the transition probability matrix of a homogeneous Markov chain, then the 2-step transition probability matrix is equal to 2P.

2. (a) What do you mean by a Markov chain and order of a Markov chain ? Defining the Markov property, show that Markov chains follows this property. 5

(b) A Markov chain $X_0, X_1, X_2, ...$ has the following transition probability matrix :

$$P = \begin{bmatrix} 0.6 & 0.3 & 0.1 \\ 0.3 & 0.3 & 0.4 \\ 0.4 & 0.1 & 0.5 \end{bmatrix}$$

Find the limiting distribution of the chain. 5

3. (a) Describe and explain the classical gambler's ruin problem in the context of stochastic processes. Also explain the meaning of 'ruin' of gambler(s). 5

(b) Suppose that customers arrive at a facility according to a Poisson process having rate $\lambda = 2$. Let $N(t)$ be the number of customers that have arrived upto time t. Determine the probability $P[N(3) = 6/N(1) = 2]$. 5

4. (a) Explain, what is a renewal process. In what sense, it is a generalization of a classical Poisson process ? 5

(b) Given that the autocorrelation function for a stationary ergodic process with no periodic component is

$$R_{XX}(\tau) = 25 \frac{4}{1 + 6\tau^2}$$

Find the mean and variance of the process $\{X(t)\}$. 5

5. (a) Arrivals at a telephone booth are considered to follow Poisson distribution with an average time of

12 minutes between one arrival and the next. The length of a phone call is assumed to be distributed exponentially with mean 4 minutes. Then : 7

(i) Find the average number of persons waiting in the system.

(ii) What is the probability that a person arriving at the booth will have to wait in the queue ?

(iii) What is the probability that it will take him more than 10 minutes altogether to wait for the phone and complete his call ?

(iv) Estimate the fraction of the day when the phone will be in use.

(v) What is the average length of the queue that forms from time to time ?

(b) What are the characteristics of a queuing system ? 3

6. (a) Define a Wiener process and derive the mean and variance of the Wiener process. 5

(b) A symmetric random walk starts at $x = 0$. Find the probabilities that walk is at : 5

(i) X = 0 after 10 steps;

(ii) X = 1 after 5 steps; and

(iii) X = – 3 after 9 steps.

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