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BTPS–021

INTEGRATED TEACHER EDUCATION PROGRAMME (ITEP) (BSCBED)

Term-End Examination

June, 2026

BTPS-021 : LINEAR ALGEBRA

Time : 3 Hours *Maximum Marks : 100*

Note : *(i) Question No. **1** is compulsory.*

*(ii) Attempt any **four** questions from Q. No. **2** to Q. No. **7**.*

(iii) Use of calculator is not allowed.

(iv) Symbols have their usual meanings.

1. Which of the following statements are True *or* False ? Give a short proof or a counter-example as justification : 10×2=20

   (i) Every finite ring is a field.

(ii) If $u \cdot v = |u|\,|v|$, then $u = v$.

(iii) The set $\{(x, y) \in \mathbf{R}^2 : x \geq 0, y \geq 0\}$ is a subspace of $\mathbf{R}^2$.

(iv) Every finite dimensional vector space has infinitely many bases.

(v) Every invertible linear transformation has rank equal to nullity.

(vi) Rank (T) + Nullity (T) is equal to the dimension of thecodomain.

(vii) An $n \times n$ matrix with rank n is invertible.

(viii) If AB = BA then $|A + B| = |A| + |B|$.

(ix) The product of eigenvalues of a matrix is equal to its determinant.

(x) A matrix with repeated eigenvalues can never be diagonalizable.

2. (a) Given the sets : 10

$$A = \{x \in \mathbf{Z} : -5 \leq x \leq 5\}$$

$$B = \{x \in \mathbf{Z} : x^2 < 16\}$$

(i) Write A and B in Roster form.

(ii) Find $A \cup B$, $A \cap B$ and $A - B$.

(iii) Represent the result using a Venn diagram.

(b) Check whether the operation * defined by $a * b = a + b + ab$, where $a, b \in \mathbf{R}$ is a binary operator on $\mathbf{R}$. 5

(c) Find the vector equation of the line passing through the point (1, 2, 3) and parallel to the vector (2, – 1, 4). 5

3. (a) Find the equation of the plane passing through the point (1, 2, 3) and perpendicular to the vector (2, – 1, 4). 5

(b) Which of the following subsets of $\mathbf{R}^3$ are vector spaces over $\mathbf{R}$? 10

(i) $W_1 = \{(x, y, z) : x + y + z = 0\}$

(ii) $W_2 = \{(x, y, z) : xy = 0\}$

(iii) $W_3 = \{(x, y, z) : x^2 + y^2 = z^2\}$

(iv) $W_4 = \{(x, y, z) : x + y + z \geq 0\}$

(v) $W_5 = \{(x, y, z) : x = \sin y\}$

(c) Determine which of the following sets are linearly independent : 5

(i) $\{(1,2,3),(2,4,6),(1,1,1)\}$

(ii) $\{1+x, x+x^2, 1+x^2\}$

(iii) $\{1, x, x^2, x^2+x+1\}$

(iv) $\left\{\sin x, \cos x, \sin\left(x+\frac{\pi}{2}\right)\right\}$

4. (a) Show that the set $\{(1,0,1),(2,1,3),(3,1,4)\}$ is a basis of $\mathbf{R}^3$. Express the vector (5, 2, 7) as a linear combination of these vectors. 6

(b) Let $T : \mathbf{R}^3 \to \mathbf{R}^3$ be defined by :

$T(x,y,z) = (x+y, y+z, x-z)$.

Find : 10

(i) The matrix of T w.r.t. the standard basis

(ii) Ker T

(iii) rank T

(iv) R(T)

(v) Nullity (T)

Verify the Rank-Nullity Theorem.

(c) Reduce the matrix : 4

$$A = \begin{bmatrix} 1 & 2 & 1 & 3 \\ 2 & 4 & 3 & 7 \\ 1 & 3 & 2 & 4 \\ 3 & 8 & 5 & 11 \end{bmatrix}$$

to row-echelon form. Hence, determine the rank of A.

5. (a) Evaluate the determinant using determinant properties only : 5

$$\begin{vmatrix} a+b+c & a & a \\ b & a+b+c & b \\ c & c & a+b+c \end{vmatrix}$$

Hence find the values of a,b,c for which the determinant is 0.

(b) Solve the following system using Cramer's rule : 5

$$x + y + z = 6$$

$$2x + 3y - z = 5$$

$$x - 2y + 4z = 8 .$$

(c) Find whether the following matrix is singular or non-singular. If non-singular, find its inverse : 6

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 5 & 7 \\ 1 & 0 & 8 \end{bmatrix}$$

(d) Prove that the determinant of a triangular matrix is equal to the product of diagonal elements. 4

6. (a) Find the eigenvalues and eigenvectors of : 5

$$A = \begin{bmatrix} 4 & 1 & 0 \\ 1 & 4 & 1 \\ 0 & 1 & 4 \end{bmatrix}.$$

(b) Obtain the characteristic polynomial and eigenvectors of : 6

$$A = \begin{bmatrix} 2 & 1 & 1 \\ 0 & 3 & 2 \\ 0 & 0 & 5 \end{bmatrix}$$

(c) Diagonalize the matrix $A = \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$ and the matrix P such that $P^{-1}AP = D$. 5

(d) If A is invertible and λ is an eigen value of A, prove that $1/\lambda$ is an eigen value of A^{-1}. 4

7. (a) For $A = \begin{bmatrix} 4 & 1 & 0 \\ 0 & 4 & 1 \\ 0 & 0 & 4 \end{bmatrix}$, find : 6

(i) Characteristic polynomial

(ii) Minimal polynomial

(iii) Verify Cayley-Hamilton theorem

(b) If the characteristic polynomial of a matrix is $f(t) = (t-2)^3(t+1)^2$, find all possible minimal polynomials and justify your answer. 4

(c) Find A^{-1} using Cayley-Hamilton theorem : 5

$$A = \begin{bmatrix} 3 & 1 & 1 \\ 0 & 2 & 1 \\ 0 & 0 & 1 \end{bmatrix}.$$

(d) A matrix A satisfies the equation :

$$A^3 - 6A^2 + 11A - 6I = 0$$

Find the possible eigenvalues of A. If A is a 3 × 3 matrix, determine its characteristic polynomial and minimal polynomial. 5

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