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POST GRADUATE DIPLOMA IN APPLIED STATISTICS (PGDAST)

Term-End Examination

June, 2026

MST-003 : PROBABILITY THEORY

Time : 3 Hours *Maximum Marks : 50*

Note : *(i) Question No.* ***1*** *is compulsory.*

(ii) Attempt any ***four*** *questions from the remaining Question Nos.* ***2*** *to* ***7****.*

(iii) Use of scientific calculator (non-programmable) is allowed.

(iv) Use of Formulae and Statistical Tables Booklet for PGDAST is allowed.

(v) Symbols have their usual meanings.

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1. State whether the following statements are True *or* False. Give reasons in support of your answers : 5×2=10

(a) If A and B are independent events, then all of the following are true :

(i) $P(A \cap B) = P(A)P(B)$

(ii) $P(A/B) = P(A)$

(iii) $P(B/A) = P(B)$

(iv) $P(A \cup B) = 1 - P(\overline{A}) - P(\overline{B})$

(b) For any two random variables X and Y, we always have $E(XY) = E(X)E(Y)$.

(c) If X is a discrete random variable such that $P(X > s + t \,/\, X > s) = P(X > t)$ then X follows hypergeometric distribution.

(d) If f_X denotes probability density function of a continuous random variable, then $f_X(x) \leq 1 \; \forall x$ always holds.

(e) If $P(\overline{A}) = \frac{2}{3}$ and $A \cup B = s$, $A \cap B = \phi$ then $P(\overline{B}) = \frac{1}{3}$.

2. (a) Three light bulbs are chosen at random from 15 bulbs of which 5 are defective. Find the probability that :
   (i) None is defective.
   (ii) Exactly one is defective.
   (iii) At least one is defective.

   (b) A box contains 6 red, 4 white and 5 black balls. A person draws 4 balls from the box at random. Find the probability that among the balls drawn there is at least one ball of each colour.

   (c) If A and B are independent events, then prove that $\overline{A}$ and B are also independent. 4+4+2

3. Suppose three balls b_1, b_2, b_3 be placed randomly in three cells. Write all possible outcomes of placing the three balls in three

cells. If X denotes the number of balls in cell I and Y be the number of cells occupied, write joint Probability Mass Function (PMF) of bivariate random variable (X, Y). Also, write marginal PMF's of random variables X and Y. Write conditional PMF of the random variable X given Y = 2. 10

4. Four coins were tossed and number of heads noted. The experiment is repeated 200 times. The distribution of number of tosses showing 0, 1, 2, 3 and 4 are given as follows :

Number of heads	Number of tosses
0	15
1	35
2	90
3	40
4	20

Fit a binomial distribution to these observed results assuming that the nature of the coin is not known. 10

5. (a) In a certain city, the daily consumption of electric power in millions of kilowatt-hours can be treated as a random variable having a Gamma distribution with parameters $\lambda = \frac{1}{2}$ and $r = 3$. If the power plant of this city has a daily capacity of 12 million kilowatt-hours, what is the probability that this power supply will be inadequate on any given day ? 5

(b) The marks obtained by the students in Mathematics in an examination are normally distributed with mean 150 and standard deviation 14.14. Find the probability that a student selected at random secured a total of (i) 180 or above and (ii) 135 or less. 5

6. (a) In a factory there are three machines X, Y and Z which produce 20%, 30%, 50% items respectively. Past experience

shows that percentage of defective items produced by machines X, Y, Z and 10%, 5%, 2% respectively. An item from the production of these machines is selected at random and it is found defective. What is the probability that it is produced by machine X. 5

(b) Joint density function of a two-dimensional random variable (X, Y) is given by

$$f_{X,Y}(x, y) = \begin{cases} x + y, & 0 \le x \le 1, 0 \le y \le 1 \\ 0, & \text{otherwise} \end{cases}$$

Find the conditional density function of Y given X. 5

7. If a random variable X follows geometric distribution with parameter p, then find expected value of X and variance of X. 10

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